SANTIAGO NUMÉRICO II

Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Facultad de Matemáticas, Pontificia Universidad Católica de Chile
Santiago, Diciembre 09 - 11, 2010

PROGRAMA y RESÚMENES

Índice

1. Introducción

El **Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales** ha sido organizado en conferencias secuenciales de **45 y 30 minutos** de duración (40 y 25 minutos de exposición, respectivamente, y 5 minutos para preguntas y comentarios). Todas las charlas se llevarán a cabo en el Auditorio Ninoslav Bralić de la Facultad de Matemáticas.

En las páginas siguientes se detalla primero la programación correspondiente y luego se incluyen los resúmenes de cada uno de los trabajos (en orden alfabético, según autores). Cuando hay más de un autor, aquel que aparece subrayado corresponde al expositor.

Los organizadores expresamos nuestro agradecimiento a los auspiciadores que se indican a continuación, los cuales han aportado gran parte de los recursos necesarios para el financiamiento de este evento:

- Centro de Modelamiento Matemático (CMM) de la Universidad de Chile,
- Facultad de Matemáticas de la Pontificia Universidad Católica de Chile,
- Centro de Investigación en Ingeniería Matemática (CI^2MA) de la Universidad de Concepción, y
- Facultad de Ciencias Físicas y Matemáticas de la Universidad de Concepción.

Igualmente, extendemos nuestro reconocimiento y gratitud a todos los expositores, quienes han hecho posible la realización de **Santiago Numérico II**.

Comité Organizador

Santiago, Diciembre 2010

2. Jueves, 9 de Diciembre

8.30-9.15 INSCRIPCIÓN

9.15-9.30 BIENVENIDA DEL DECANO

[*Moderador*: N. HEUER]

9.30-10.15 Pavel Bochev, Denis Ridzal, Guglielmo Scovazzi, Mikhail Shashkov: *Optimization-based computational modeling, or how to achieve better predictiveness with less complexity.*

10.15-10.45 Ralf Hiptmair, Carlos Jerez-Hanckes: *Multiple traces boundary integral formulation for Helmholtz transmission problems.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Fernando Betancourt, Raimund Bürger, Kenneth H. Karlsen, Elmer M. Tory: *On nonlocal conservation laws modeling sedimentation.*

11.45-12.15 Gabriel Acosta, Gabriela Armentano: *Convergence of the FEM in domains with external cusps.*

12.15-12.45 Pablo Castañeda, Dan Marchesin, Frederico Furtado: *Riemann solution for the three-phase injection problem in virgin reservoirs with general relative permeabilities.*

12.45-15.00 ALMUERZO

[*Moderador*: S. GUTIÉRREZ]

15.00-15.45 Ernst P. Stephan: *Solution procedures for frictional contact.*

15.45-16.15 Tomás P. Barrios, Edwin M. Behrens, María González: *A posteriori error estimates based on the Ritz projection of the error for an augmented mixed FEM in plane linear elasticity.*

16.15-16.45 Rodolfo Rodríguez, Pablo Venegas: *Numerical approximation of the spectrum of the curl operator.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Mark Ainsworth, Alejandro Allendes, Gabriel Barrenechea, Richard Rankin: *Computable error bounds for the Fortin–Soulie and stabilized finite element methods of the Stokes problem.*

17.45-18.15 Carlos E. Zambra, Nelson O. Moraga: *Coupled heat and mass transfer in porous media with biological and chemical reactions.*

18.15-18.45 Gabriel N. Gatica, Antonio Márquez, Manuel A. Sánchez: *A velocity-pseudostress formulation for a class of quasi-Newtonian Stokes flows.*

19.30 COCKTAIL DE BIENVENIDA

3. Viernes, 10 de Diciembre

[*Moderador*: R. BÜRGER]

9.30-10.15 Francisco–Javier Sayas: *Energy estimates in semidiscrete time–domain boundary integral equations.*

10.15-10.45 Mauricio Sepúlveda: *Stabilization of a second order scheme for a GKdV-4 equation modelling surface water waves.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Eleuterio F. Toro, Annunziato Siviglia: *A model for blood flow in vessels with discontinuous material properties: exact solutions and numerical methods.*

11.45-12.15 Jessika Camaño, Rodolfo Rodríguez: *Well-posedness and error estimates of a hybrid formulation for the eddy current problem.*

12.15-12.45 Sergio Gutiérrez: *About two problems in structural optimization that can be approximately solved using homogenization.*

12.45-15.00 ALMUERZO

[*Moderador*: E. TORO]

15.00-15.45 Todd Arbogast: *Multiscale mortar methods for flow in heterogeneous porous media.*

15.45-16.15 Mónica Selva-Soto, Caren Tischendorf: *A coupled system of partial differential and differential algebraic equations for modeling electrical circuits.*

16.15-16.45 Mauricio A. Barrientos, Matthias Maischak: *A dual-mixed analysis for incompressible quasi-Newtonian flows.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Stefan Berres, Ricardo Ruiz-Baier: *A fully adaptive numerical approximation for a two-dimensional epidemic model with nonlinear cross-diffusion.*

17.45-18.15 Jessika Camaño, Gabriel N. Gatica, Ricardo Oyarzúa, Pablo Venegas: *Analysis of an augmented mixed finite element method for the Stokes-Darcy coupled problem.*

18.15-18.45 Jeffrey Zitelli, Ignacio Muga, Leszek Demkowicz, Jayadeep Gopalakrishnan, David Pardo, Victor M. Calo: *A class of discontinuous Petrov-Galerkin methods: the optimal test norm and time-harmonic wave propagation.*

20.30 CENA DE CAMARADERÍA
(**Restaurant MESÓN NERUDIANO**)

4. Sábado, 11 de Diciembre

[*Moderador*: G. GATICA]

9.30-10.00	Gabriela Armentano, Claudio Padra, Rodolfo Rodríguez, Mario Scheble: *An hp finite element adaptive method for fluid-solid interactions.*
10.00-10.30	Rommel Bustinza, Francisco-Javier Sayas: *An a priori error analysis of the local discontinuous Galerkin method for Signorini type problems.*
10.30-11.00	Nelson O. Moraga: *Fluid dynamics and unsteady heat transfer in non–Newtonian liquid solidification with natural convection inside cavities.*
11.00-11.30	COFFEE BREAK
11.30-12.00	Alfredo Bermúdez, Bibiana López-Rodríguez, Rodolfo Rodríguez, Pilar Salgado: *Numerical solution of transient eddy current problems with input current intensities as boundary data.*
12.00-12.30	Ariel A. Ibarra, Adrián P. Cisilino: *Numerical homogenization of trabecular bone microstructure using the standard mechanics approach.*
12.30-13.00	Franz Chouly, Norbert Heuer: *A Nitsche-based domain decomposition for hypersingular integral equations.*
13.00-15.00	ALMUERZO

5. Resúmenes

Convergence of the FEM in domains with external cusps *

Gabriel Acosta † Gabriela Armentano‡

Abstract

In [1] the finite element method was applied to a non-homogeneous Neumann problem on a cuspidal domain $\Omega \subset \mathbb{R}^2$, and using regularity results developed in [2], quasi-optimal order error estimates in the energy norm were obtained for certain graded meshes. In this talk we present similar results for the error in the L^2 norm. Since many classical results in the theory of Sobolev spaces do not apply to the domain under consideration, our estimates require a particular duality treatment working on appropriate weighted spaces. Our talk is based on the recent article [3].

References

[1] G. Acosta, M. G. Armentano, R. G. Durán and A. L. Lombardi, Finite element approximations in a non-Lipschitz domain, SIAM Journal on Numerical Analysis **45**(1), 277-295 (2007).

[2] G. Acosta, M. G. Armentano, R. G. Durán and A. L. Lombardi, Nonhomogeneous Neumann problem for the Poisson equation in domains with an external cusp, Journal of Mathematical Analysis and Applications **310**(2), 397-411 (2005).

[3] G. Acosta, M. G. Armentano, Finite element approximations in a non-Lipschitz domain: Part II, To Appear in Math. Comp.

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SANTIAGO NUMÉRICO II
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Computable error bounds for the Fortin–Soulie and stabilized finite element methods of the Stokes problem *

Mark Ainsworth † Alejandro Allendes ‡ Gabriel Barrenechea §
Richard Rankin ¶

Abstract

We propose computable a posteriori error estimates for stabilization of lower–order mixed finite elements and the Fortin–Soulie finite element approximations of the Stokes problem. The estimator is completely free of unknown constants and gives a guaranteed numerical upper bound on the error, in terms of a lower bound for the inf-sup constant of the underlying continuous problem. The estimator is also shown to provide a lower bound on the error up to a constant and higher order data oscillation terms. Numerical results are presented illustrating the theory and the performance of the estimator.

References

[1] Mark Ainsworth and J.T. Oden, *A Posteriori Error Estimation in Finite Element Analysis.* Pure and Applied Mathematics, Wiley–Interscience. Jhon Wiley & Sons, New York, 2000.

[2] Mark Ainsworth and Richard Rankin, *Fully Computable bounds for the error in nonconforming finite element approximations of arbitrary order on triangular elements.* SIAM J. Numer. Anal., 46:3207–3232, 2008.

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[3] Gabriel Barrenechea G. and Frédéric Valentin, *Consistent Local Projection Stabilized Finite Element Methods.* Submitted to SIAM J. NUMER. ANAL.

[4] W. Dörfler and M. Ainsworth, *Reliable a posteriori error control for nonconforming finite element approximation of stokes flow.* Math. Comput., 74:1599–1616, 2005.

[5] Pavel Bochev and Max Gunzburger, *An absolute stable Pressure-Poisson stabilized finite element method for the Stokes Equations.* SIAM J. NUMER. ANAL. Vol. 42, No. 3, pp. 1189–1207

[6] Hans–Görg Roos, Martin Stynes and Lutz Tobiska, *Robust Numerical Methods for Singularly Perturbed Differential Equations.* Springer Series in Computational Mathematics, Springer Berlin Heidelberg, Volume 24, 2008.

[7] G. Stoyan, *Towards discrete velte decompositions narrow bounds for the inf-sup constants.* Comp. Math. Appl., 38:243–261, 1999.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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Multiscale mortar methods for flow in heterogeneous porous media*

TODD ARBOGAST †

Abstract

We consider a second order elliptic problem with a heterogeneous coefficient written in mixed form. We view the domain decomposition method as a multiscale method with restricted degrees of freedom on the interfaces. We devise an effective but purely local multiscale method that incorporates information from homogenization theory. We also use this decomposition method approach to devise effective preconditioners that incorporate exact coarse-scale information to iteratively solve the full fine-scale problem.

References

[1] T. ARBOGAST, L. C. COWSAR, M. F. WHEELER, AND I. YOTOV. Mixed finite element methods on non-matching multiblock grids. *SIAM J. Numer. Anal.*, 37:1295–1315, 2000.

[2] T. ARBOGAST, G. PENCHEVA, M. F. WHEELER, AND I. YOTOV. A multiscale mortar mixed finite element method. *Multiscale Model. Simul.*, 6(1):319–346, 2007.

[3] T. ARBOGAST. Homogenization-based mixed multiscale finite elements for problems with anisotropy. *Submitted*, 2010.

[4] T. ARBOGAST. Mixed multiscale methods for heterogeneous elliptic problems, in Numerical Analysis of Multiscale Problems, Springer, 2011 (expected).

[5] T. ARBOGAST AND HAILONG XIAO. A multiscale mortar mixed space based on homogenization for heterogeneous elliptic problems. *In preparation*, 2010.

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SANTIAGO NUMÉRICO II
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An *hp* finite element adaptive method for fluid-solid interactions *

Gabriela Armentano † Claudio Padra‡
Rodolfo Rodríguez § Mario Scheble ¶

Abstract

In this paper we propose an *hp* finite element method to solve a two-dimensional fluid-structure vibration problem. This problem arises from the computation of the vibration modes of a bundle of parallel tubes immersed in an incompressible fluid. We use a residual-type a posteriori error indicator to guide an *hp* adaptive algorithm. Since the tubes are allowed to be different, the weak formulation is a non-standard generalized eigenvalue problem. This feature is inherited by the algebraic system obtained by the discretization process. We introduce an algebraic technique to solve this particular spectral problem. We report several numerical tests which allow us to assess the performance of the scheme.

References

[1] Armentano MG, Padra C. A posteriori error estimates for the Steklov eigenvalue problem. *Applied Numerical Matematics* 2008; **58**:593–601.

[2] Armentano MG, Padra C, Rodríguez R, Scheble M. An *hp* finite element adaptive scheme to solve the Laplace model for fluid-solid vibrations. Preprint DIM 2009-19, Universidad de Concepción, Concepción, 2009, to appear in CMAME.

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[3] Bermúdez A, Rodríguez R, Santamarina D. A finite element solution of an added mass formulation for coupled fluid-solid vibrations. *Numerische Mathematik* 2000; **87**:201–227.

[4] Conca C, Planchard J, Vanninathan M. *Fluid and Periodic Structures*, Wiley: Chichester, 1995.

[5] Melenk JM and Wohlmuth BI. On residual-based a posteriori error estimation in *hp*-FEM. *Advances in Computational Mathematics* 2001; **15**:311–331.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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A dual-mixed analysis for incompressible quasi-Newtonian flows*

Mauricio A. Barrientos † Matthias Maischak ‡

Abstract

We consider the coupling of dual-mixed finite element method and boundary integral equation method to solve a transmission problem between a lineal Stokes flow with a quasi-Newtonian flow with mixed boundary conditions . The result is a new mixed scheme for the quasi-Newtonian problem. The approach is based on the introduction of both the flux and the strain tensor as futher unknows, wich yields a two-fold saddle point operator equation as the resulting variational formulation. We derive existence and uniqueness of solution for the continuous and discrete formulations and provide the associated error analysis. In particular, the corresponding Galerkin scheme is defined by using piecewise constant functions and Raviar-Thomas spaces of lowest order. Most of our analysis makes use of an extension of the classical Babuska-Brezzi theory to a class of nonlinear saddle-point problems. Also, we develop a-posteriori error estimates (based on Bank-Weiser type) and propose and reliable adaptive algorithm to compute the finite elements solutions. Finally, several numerical results are provided.

References

[1] M.A. Barrientos, G.N. Gatica and N. Heuer. *An a-posteriori error estimate for a linear-nonlinear transmission problem in plane elastostatics.* Calcolo, 39 (2002), pp. 61-86.

[2] M.A. Barrientos and M. Maischak. *A-posteriori error analysis to the exterior Stokes problem.* Submitted.

[3] G.N. Gatica, M. González and S. Meddahi, *A low-order mixed finite element method for a class of quasi-Newtonian Stokes flows. Part I: a-priori error analysis.* Computer Methods in Applied Mechanics and Engineering, 193 (2004), pp. 893-911.

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[4] G.N. Gatica, M. González and S. Meddahi, *A low-order mixed finite element method for a class of quasi-Newtonian Stokes flows. II. A posteriori error analysis.* Comput. Methods Appl. Mech. Engrg., 193(9-11) (2004), pp. 893–911.

[5] G.N. Gatica and N. Heuer. *A dual-dual formulation for the coupling of mixed-FEM and BEM in hyperelasticity.* SIAM J. Numer. Anal., 38(2) (2000), pp. 380-400.

[6] G. Hsiao and W. Wendland, Boundary Integral Equations, Applied Mathematical Sciences vol. 144, Springer-Verlag Berlin Heidelberg, 2008.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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A posteriori error estimates based on the Ritz projection of the error for an augmented mixed FEM in plane linear elasticity

Tomás P. Barrios * Edwin M. Behrens † María González ‡

Abstract

We consider the augmented mixed finite element methods introduced in [2] and [3] for the problem of linear elasticity in the plane with different boundary conditions. We follow [1] and use the Ritz projection of the error to obtain a posteriori error estimates that are reliable and efficient, but that involve a non-local term. A slightly modified analysis allows us to deduce reliable, fully local a posteriori error estimates that are efficient up to those elements where we impose a boundary condition weakly. These new a posteriori error estimates are cheap and easy to implement since they do not involve any jump along the sides of the grid. Numerical experiments illustrate the performance of these a posteriori error estimates and support the theoretical results.

References

[1] Barrios, T.P. and Gatica, G.N., *An augmented mixed finite element method with Lagrange multipliers: A priori and a posteriori error analyses.* J. Comput. Appl. Math. vol. 200, pp. 653-676, (2007).

[2] Gatica, G.N., *Analysis of a new augmented mixed finite element method for linear elasticity allowing* $\mathbb{RT}_0$-$\mathbb{P}_1$-$\mathbb{P}_0$ *approximations.* M2AN Math. Model. Numer. Anal. vol. 40, pp. 1-28, (2006).

[3] Gatica, G.N., *An augmented mixed finite element method for linear elasticity with non-homogeneous Dirichlet conditions.* Electron. Trans. Numer. Anal. vol. 26, pp. 421-438, (2007).

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SANTIAGO NUMÉRICO II
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Numerical solution of transient eddy current problems with input current intensities as boundary data

Alfredo Bermúdez * Bibiana López-Rodríguez †
Rodolfo Rodríguez† Pilar Salgado*

Abstract
The aim of this talk is to analyze a numerical method to solve transient eddy current problems with input current intensities as data, formulated in terms of the magnetic field in a bounded domain including conductors and dielectrics ([1]). To this end, we introduce a time-dependent weak formulation and prove its well-posedness ([2]). Under appropriate hypotheses on the input current intensities, following [3] we show that the weak solution has additional regularity and satisfies strong forms of the equations. We propose a finite element method for space discretization based on Nédélec edge elements on tetrahedral mesh, for which we prove well-posedness and error estimates. Furthermore, we introduce an implicit Euler scheme for time discretization and prove error estimates for the fully discrete problem. Moreover, a magnetic scalar potential is introduced to deal with the curl-free condition in the dielectric domain. This approach leads to an important saving in computational effort. Finally, the method is applied to solve two problems: a test with a known analytical solution and an application to electromagnetic forming.

References

[1] Bermúdez, A., Rodríguez, R., & Salgado, P. *Numerical solution of eddy current problems in bounded domains using realistic boundary conditions.* Computer Methods in Applied Mechanics and Engineering, vol. 194, 2-5, pp. 411–426, (2005).

[2] Dautray, R. & Lions, J-L. Mathematical Analysis and Numerical Methods for Science and Technology, Volume 5, Evolution Problems I. Springer–Verlag, Berlin, 1992.

[3] Evans, L.C. Partial Differential Equations. American Mathematical Society, Providence, RI. 1998.

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SANTIAGO NUMÉRICO II
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A fully adaptive numerical approximation for a two-dimensional epidemic model with nonlinear cross-diffusion*

Stefan Berres † Ricardo Ruiz-Baier ‡

Abstract

An epidemic model is formulated by a reaction-diffusion system where the spatial pattern formation is driven by cross-diffusion. Whereas the reaction terms describe the local dynamics of susceptible and infected species, the diffusion terms account for the spatial distribution dynamics. For both self-diffusion and cross-diffusion nonlinear constitutive assumptions are suggested. To simulate the pattern formation two finite volume formulations are proposed, which employ a conservative and a non-conservative discretization, respectively. An efficient simulation is obtained by a fully adaptive multiresolution strategy. Numerical examples illustrate the impact of the cross-diffusion on the pattern formation.

References

[1] S. Berres, R. Bürger, H. Frid, *Neumann problems for quasi-linear parabolic systems modeling polydisperse suspensions*, SIAM J. Math. Anal. 38 (2006) 557–573.

[2] M. Bendahmane, M. Sepúlveda,*Convergence of a finite volume scheme for nonlocal reaction-diffusion systems modelling an epidemic disease*, Disc. Cont. Dynam. Syst. - Series B 11 (2009) 823–853.

[3] M.J. Castro, J.M. Gallardo, C. Parés, *High-order finite volume schemes based on reconstruction of states for solving hyperbolic systems with nonconservative products. Applications to shallow-water systems*, Math. Comp. 75 (2006) 1103–1134.

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SANTIAGO NUMÉRICO II
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On nonlocal conservation laws modeling sedimentation*

Fernando Betancourt † Raimund Bürger ‡
Kenneth H. Karlsen § Elmer M. Tory¶

Abstract

The well-known kinematic sedimentation model by Kynch states that the settling velocity of small equal-sized particles in a viscous fluid is a function of the local solids volume fraction. This assumption converts the one-dimensional solids continuity equation into a scalar, nonlinear conservation law with a non-convex and local flux. The present work deals with a modification of this model, and is based on the assumption that either the solids phase velocity or the solid-fluid relative velocity at a given position and time depends on the concentration in a neighborhood via convolution with a symmetric kernel function with finite support. This assumption is justified by theoretical arguments arising from stochastic sedimentation models, and leads to a conservation law with a nonlocal flux. The alternatives of velocities for which the nonlocality assumption can be stated lead to different algebraic expressions for the factor that multiplies the nonlocal flux term. In all cases, solutions are in general discontinuous and need to be defined as entropy solutions. An entropy solution concept is introduced, jump conditions are derived and uniqueness of entropy solutions in shown. Existence of entropy solutions is established by proving convergence of a difference-quadrature scheme. It turns out that only for the assumption of nonlocality for the relative velocity it is ensured that solutions of the nonlocal equation assume physically relevant solution values between zero and one. Numerical examples illustrate the behaviour of entropy solutions of the nonlocal equation.

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References

[1] Bürger, R., Karlsen, K.H., Tory, E.M. and Wendland, W.L., Model equations and instability regions for the sedimentation of polydisperse suspensions of spheres. *ZAMM Z. Angew. Math. Mech.*, 82:699–722, 2002.

[2] Bürger, R. and Tory, E.M., On upper rarefaction waves in batch settling. *Powder Technol.*, 108:74–87, 2000.

[3] Kurganov, A. and Polizzi, A., Non-oscillatory central schemes for traffic flow models with Arrhenius look-ahead dynamics. *Netw. Heterog. Media*, 4:431–451, 2009.

[4] Kynch, G.J., A theory of sedimentation. *Trans. Faraday Soc.*, 48:166–176, 1952.

[5] Richardson, J.F. and Zaki, W.N., Sedimentation and fluidization: Part I. *Trans. Instn Chem. Engrs (London)* 32:35–53, 1954.

[6] Siano, D.B., Layered sedimentation in suspensions of monodisperse spherical colloidal particles. *J. Colloid Interface Sci.* 68:111–127, 1979.

[7] Sjögreen, B., Gustavsson, K. and Gudmundsson, R., A model for peak formation in the two-phase equations. *Math. Comp.*, 76:1925–1940, 2007.

[8] Zumbrun, K., On a nonlocal dispersive equation modeling particle suspensions. *Quart. Appl. Math.*, 57:573–60, 1999.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Optimization-based computational modeling, or how to achieve better predictiveness with less complexity*

Pavel Bochev † Denis Ridzal ‡ Guglielmo Scovazzi §
Mikhail Shashkov ¶

Abstract

Discretization converts infinite dimensional mathematical models into finite dimensional algebraic equations that can be solved on a computer. This process is accompanied by unavoidable information losses which can degrade the predictiveness of the discrete equations. Compatible and regularized discretizations control these losses directly by using suitable field representations and/or by modifications of the variational forms. Such methods excel in controlling "structural" information losses responsible for the stability and well-posedness of the discrete equations. However, direct approaches become increasingly complex and restrictive for multi-physics problems comprising of fundamentally different mathematical models, and when used to control losses of "qualitative" properties such as maximum principles, positivity, monotonicity and local bounds preservation. In this talk we show how optimization ideas can be used to control externally, and with greater flexibility, information losses which are difficult (or impractical) to manage directly in the discretization process. This allows us to improve predictiveness of computational models, increase robustness and accuracy of solvers, and enable efficient reuse of code. Two examples will be presented: an optimization-based framework for multi-physics coupling [2], and an optimization-based algorithm for constrained interpolation (remap) [1]. In the first case, our approach allows to synthesize a robust and efficient solver for a coupled multiphysics problem from simpler solvers for its constituent components. To illustrate the scope of the approach we derive such a solver for nearly hyperbolic PDEs from standard, off-the-shelf algebraic multigrid solvers, which

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by themselves cannot solve the original equations [3]. The second example demonstrates how optimization ideas enable design of high-order conservative, monotone, bounds preserving remap and transport schemes which are linearity preserving on arbitrary unstructured grids, including grids with polyhedral and polygonal cells.

References

[1] Bochev, P., Ridzal, D., Scovazzi, G., Shashkov, M., *Formulation, analysis and numerical study of an optimization-based conservative interpolation (remap) of scalar fields for arbitrary Lagrangian-Eulerian methods.* Journal of Computational Physics. Submitted (2010).

[2] Bochev, P., Ridzal, D., *An optimization-based approach for the design of robust solution algorithms*, SIAM J. Num. Anal. 47 (2009) 3938–3955.

[3] Bochev, P., Ridzal D., *Additive Operator Decomposition and Optimization–Based Reconnection with Applications.* Proceedings of LSSC 2009, I. Lirkov, S. Margenov and J. Wasniewski (Eds.): LSSC 2009, Springer Lecture Notes in Computer Science, LNCS 5910, (2010).

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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An a priori error analysis of the local discontinuous Galerkin method for Signorini type problems *

Rommel Bustinza † Francisco-Javier Sayas ‡

Abstract

In this talk we propose and analyze a local discontinuous Galerkin method for an elliptic variational inequality of the first kind that corresponds to a Poisson equation with Signorini type condition on part of the boundary. The method uses piecewise polynomials of degree one for the field variable and of degree zero or one for the approximation of its gradient. We show optimal convergence for the method and illustrate it with some numerical experiments.

References

[1] D.N. Arnold, F. Brezzi, B. Cockburn, and L.D. Marini: *Unified analysis of discontinuous Galerkin methods for elliptic problems.* SIAM Journal on Numerical Analysis, vol. 39, 5, pp. 1749-1779, (2001).

[2] K. Atkinson, and W. Han: *Theoretical Numerical Analysis. A functional analysis framework.* Springer-Verlag, New York, (Second edition), 2005.

[3] R. Bustinza, G.N. Gatica, and F.-J. Sayas: *On the coupling of local discontinuous Galerkin and boundary element methods for nonlinear exterior transmission problems.* IMA Journal of Numerical Analysis, vol. 28, pp. 225-244, (2008).

[4] P. Castillo, B. Cockburn, I. Perugia, and D. Schötzau: *An a priori error analysis of the local discontinuous Galerkin method for elliptic problems.* SIAM Journal on Numerical Analysis, vol. 38, 5, pp. 1676-1706, (2000).

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SANTIAGO NUMÉRICO II
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Analysis of an augmented mixed finite element method for the Stokes-Darcy coupled problem*

Jessika Camaño † Gabriel N. Gatica ‡
Ricardo Oyarzúa § Pablo Venegas ¶

Abstract

In this paper we analyze an augmented mixed finite element method for the coupling of fluid flow with porous media flow. Flows are governed by the Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider a semi-augmented mixed formulation, augmented in the Stokes domain and dual-mixed in the Darcy region, which yields a compact perturbation of an invertible mapping as the resulting operator equation. The approach, which extends recent results on the a priori and a posteriori error analysis of a fully-mixed formulation for the Stokes-Darcy model, is based on the introduction of the Galerkin least-squares type terms arising from the constitutive and equilibrium equations of the Stokes equation, and from the relations defining the free fluid pressure in terms of the stress tensor and the vorticity in terms of the free fluid velocity. All these terms are multiplied by stabilization parameters that can be chosen so that the resulting continuous formulation becomes well posed. We then apply a classical result on projection methods for Fredholm operators of index zero to show, under suitable hypotheses on the finite element subspaces for the Darcy region, that the use of arbitrary finite element subspaces for the Stokes domain implies the well-posedness of the corresponding augmented Stokes-Darcy Galerkin scheme. Next, we derive a reliable and efficient residual-based a posteriori error estimator for the augmented mixed finite element scheme.

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References

[1] I. Babuška and G.N. Gatica, *A residual-based a posteriori error estimator for the Stokes-Darcy coupled problem.* SIAM Journal on Numerical Analysis, vol. 48, 2, pp. 498-523, (2010).

[2] T. Barrios, G.N. Gatica, M. González, N. Heuer, *A residual based a posteriori error estimator for an augmented mixed finite element method in linear elasticity.* Mathematical Modelling and Numerical Analysis, vol. 40, 5, pp. 843-869, (2006).

[3] L. Figueroa, G.N. Gatica, N. Heuer, *A priori and a posteriori error analysis of an augmented mixed finite element method for incompressible fluid flows.* Computer Methods in Applied Mechanics and Engineering, vol. 198, 2, pp. 280-291, (2008).

[4] G.N. Gatica, *Analysis of a new augmented mixed finite element method for linear elasticity allowing* $\mathbb{RT}_0-\mathbb{P}_1-\mathbb{P}_0$ *approximations.* Mathematical Modelling and Numerical Analysis, vol. 40, 1, pp. 1-28, (2006).

[5] G.N. Gatica, R. Oyarzúa and F.-J. Sayas, *Analysis of fully-mixed finite element methods for the Stokes-Darcy coupled problem.* Mathematics of Computation, to appear.

[6] G.N. Gatica, R. Oyarzúa and F.-J. Sayas, *A residual-based a posteriori error estimator for a fully-mixed formulation of the Stokes-Darcy coupled problem.* Preprint 2010-12, Departamento de Ingeniería Matemática, Universidad de Concepción, (2010).

[7] G.N. Gatica, R. Oyarzúa and F.-J. Sayas, *A twofold saddle point approach for the coupling of fluid flow with nonlinear porous media flow.* Preprint 2010-19, Departamento de Ingeniería Matemática, Universidad de Concepción, (2010).

SANTIAGO NUMÉRICO II
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Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Well-posedness and error estimates of a hybrid formulation for the eddy current problem

Jessika Camaño * Rodolfo Rodríguez †

Abstract

We propose an alternative approach to introduce source current data in the eddy current problem formulated in terms of the electric field $\mathbf{E}_C$ in the conductor and the magnetic field $\mathbf{H}_I$ in the insulator. This extends previous results from [1], where the well-posedness of a similar hybrid formulation has been proved. However it is not clear how to prove thorough error estimates for the discretization of this approach. We propose an alternative auxiliary problem with suitable boundary conditions, which allows us to prove the solvability of the continuous problem and error estimates considering Nédélec finite elements to compute both, the electric and the magnetic field.

References

[1] Alonso-Rodríguez, A., Valli, A., *Eddy Current Approximation of Maxwell Equations: Theory, Algorithms and Applications.* Modeling, Simulation and Applications, Springer-Verlag Italia (2010)

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SANTIAGO NUMÉRICO II
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Riemann solution for the three-phase injection problem in virgin reservoirs with general relative permeabilities

Pablo Castañeda * Dan Marchesin † Frederico Furtado ‡

Abstract

We consider the Riemann problem for two conservation laws representing the injection of two fluids into a virgin reservoir containing a third fluid. We assume that the three fluids are immiscible and do not exchange mass. The relative permeabilities are given either by the Corey or Stone models as general saturation powers. For each proportion of the injected fluids the Riemann solution profile and the wave curves are found. The solution consists constant states, rarefaction and shock waves. Typically there are two wave groups: the slower one is composed by a rarefaction adjacent to a shock, in which the three fluid saturations change. Depending on the initial conditions, a constant state can exist between the two wave groups. In the faster wave group there are only two fluids, and it consists either of a rarefaction adjacent to a shock or of a single shock. The Riemann solution that maximizes useful oil recovery is represented by a single wave group involving the three fluids, and it consists of a rarefaction adjacent to a shock wave, leading to a shock that is faster than in the other cases. We also observe that both models give qualitatively the same solutions profiles, despite the presence of an elliptic region in Stone model. In particular we determine, for any choice of the phase viscosities and of the permeability expressions, the proportion of the injected fluids that maximizes the magnitude and speed of the principal shock, which is responsible for the recovery at breakthrough. The three-phase wave curve in this Riemann solution and the location of the umbilic point (or the elliptic region) are correlated. The wave curve in the saturation triangle and its profile are found numerically, leading to the construction of Riemann solutions for Corey and Stone models for different permeabilities. The Riemann construction of solutions for the general Corey or Stone models can be used to improve the results in oil recovery modeling. We demonstrate the well-posedness of the solutions for small parameter deviations of permeability powers or viscosities.

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References

[1] Azevedo, A. V.; de Souza, A. J.; Furtado, F.; Marchesin, D. and Plohr, B. (2009) “The solution by the wave curve method of three-phase flow in virgin reservoirs”. *Transp. Porous Med.*, **Vol. 83**, No. 1, pp. 99-125.

[2] Fayers, F. J. (1989) “Extension of Stone’s method 1 and conditions for real characteristic three-phase flow”. *SPE Reservoir Engineering*, **Vol. 4**, No. 4, pp. 437-445.

[3] Furtado, F. (1989) *Structural stability of nonlinear waves for conservation laws.* PhD Thesis, New York University.

[4] Smoller, J. (1994) *Shock waves and reaction-diffusion equations.* New York, Springer-Verlag.

SANTIAGO NUMÉRICO II
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Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

A Nitsche-based domain decomposition for hypersingular integral equations *

FRANZ CHOULY † NORBERT HEUER ‡

Abstract

We have recently analyzed several non-conforming boundary element discretizations of hypersingular boundary integral equations, namely Lagrangian multipliers for essential conditions on the boundary of open surfaces [1], Crouzeix–Raviart elements [3], and domain decomposition with so-called mortar coupling [2]. Even though none of the discrete formulations has a continuous setting (due to a missing well-posed trace operator in the corresponding energy space) they all converge quasi-optimally or almost quasi-optimally. In this talk we present a domain decomposition method with Nitsche coupling. The principal advantage of the Nitsche coupling is that it allows for symmetric linear systems. We prove almost quasi-optimal convergence of this method for hypersingular integral equations in broken Sobolev norms of order 1/2. Sub-domain decompositions can be geometrically non-conforming and meshes must be quasi-uniform only on sub-domains. Numerical results confirm the theory.

References

[1] G. N. GATICA, M. HEALEY, AND N. HEUER, *The boundary element method with Lagrangian multipliers*, Numerical Methods for Partial Differential Equations, vol. 25, pp. 1303–1319, (2009).

[2] M. HEALEY AND N. HEUER, *Mortar boundary elements*, SIAM Journal on Numerical Analysis, vol. 48, pp. 1395–1418, (2010).

[3] N. HEUER AND F.-J. SAYAS, *Crouzeix–Raviart boundary elements*, Numerische Mathematik, vol. 112, pp. 381–401, (2009).

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SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

A velocity-pseudostress formulation for a class of quasi-Newtonian Stokes flows*

Gabriel N. Gatica† Antonio Márquez‡ Manuel A. Sánchez§

Abstract

In this paper we introduce and analyze new mixed finite element schemes for a class of nonlinear Stokes models arising in quasi-Newtonian fluids. The methods are based on a non-standard mixed approach in which the velocity, the pressure, and the pseudostress are the original unknowns. However, we use the incompressibility condition to eliminate the pressure, and set the velocity gradient as an auxiliary unknown, which yields a twofold saddle point operator equation as the resulting dual-mixed variational formulation. In addition, a suitable augmented version of the latter showing a single saddle point structure is also considered. We apply known results from nonlinear functional analysis to prove that the corresponding continuous and discrete schemes are well-posed. In particular, we show that Raviart-Thomas elements of order $k \geq 0$ for the pseudostress, and piecewise polynomials of degree k for the velocity and its gradient, ensure the well-posedness of the associated Galerkin schemes. Moreover, we prove that any finite element subspace of the square integrable tensors can be employed to approximate the velocity gradient in the case of the augmented formulation. Then, we derive a reliable and efficient residual-based a posteriori error estimator for each scheme. Finally, we provide several numerical results illustrating the good performance of the resulting mixed finite element methods, confirming the theoretical properties of the estimator, and showing the behaviour of the associated adaptive algorithms.

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References

[1] P.B. Bochev and M.D. Gunzburger, *Least-squares methods for the velocity-pressure-stress formulation of the Stokes equations.* Computer Methods in Applied Mechanics and Engineering, vol. 126, 3-4, pp. 267-287, (1995).

[2] Z. Cai, Ch. Tong, P.S. Vassilevski, and Ch. Wang, *Mixed finite element methods for incompressible flow: stationary Stokes equations.* Numerical Methods for Partial Differential Equations, vol. 26, 4, pp. 957-978, (2009).

[3] C. Carstensen, *A posteriori error estimate for the mixed finite element method.* Mathematics of Computation, vol. 66, 218, pp. 465-478, (1997).

[4] V.J. Ervin, J.S. Howell, and I. Stanculescu, *A dual-mixed approximation method for a three-field model of a nonlinear generalized Stokes problem.* Computer Methods in Applied Mechanics and Engineering, vol. 197, 33-40, pp. 2886-2900, (2008).

[5] L. Figueroa, G.N. Gatica, and A. Márquez, *Augmented mixed finite element methods for the stationary Stokes Equations.* SIAM Journal on Scientific Computing, vol. 31, 2, pp. 1082-1119, (2008).

[6] G.N. Gatica, M. González, and S. Meddahi, *A low-order mixed finite element method for a class of quasi-Newtonian Stokes flows. Part I: a-priori error analysis.* Computer Methods in Applied Mechanics and Engineering, vol. 193, 9-11, pp. 881-892, (2004).

[7] G.N. Gatica, N. Heuer, and S. Meddahi, *On the numerical analysis of nonlinear twofold saddle point problems.* IMA Journal of Numerical Analysis, vol. 23, 2, pp. 301-330, (2003).

[8] G.N. Gatica, A. Márquez, and M. Sánchez, *Analysis of a velocity-pressure-pseudostress formulation for the stationary Stokes equations.* Computer Methods in Applied Mechanics and Engineering, vol. 199, 17-20, pp. 1064-1079, (2010).

[9] G.N. Gatica, A. Márquez, and M. Sánchez, *A priori and a posteriori error analyses of a velocity-pseudostress formulation for a class of quasi-Newtonian Stokes flows.* Preprint 2010-20, Departamento de Ingeniería Matemática, Universidad de Concepción, (2010).

[10] J.S. Howell, *Dual-mixed finite element approximation of Stokes and nonlinear Stokes problems using trace-free velocity gradients.* Journal of Computational and Applied Mathematics, vol. 231, 2, pp. 780-792, (2009).

[11] R. Verfürth, A Review of A Posteriori Error Estimation and Adaptive-Mesh-Refinement Techniques. Wiley-Teubner (Chichester), 1996.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

About two problems in structural optimization that can be approximately solved using homogenization*

SERGIO GUTIÉRREZ †

Abstract

The Theory of Homogenization addresses the problem of coupling different length scales, which pervades many fields like optimal design, materials science, climatology, etc. In this talk we are concerned with two problems in structural optimization: reinforcement distribution in non slender reinforced concrete elements and controlling stress concentration. The idea is to use homogenization to relax the associated optimization problems, therefore considering generalized designs that admit for the use of composite materials. However, these designs are generally too expensive, then we penalize the use of composites to come back to classical designs. The optimization problems are approximately solved using FEM to solve the state and adjoint problems and a steepest descent algorithm.

References

[1] ALLAIRE, G. AND GUTIÉRREZ, S. *Optimal Design in Small Amplitude Homogenization.* ESAIM Mathemtical Modelling and Numerical Analysis, vol. 41, 3, pp. 543-574, (2007).

[2] HERRANZ, J.P., SANTA MARÍA, H., GUTIÉRREZ, S. AND RIDDELL, R. *Optimal Strut-and-Tie models using the Full Homogenization Method.* Submitted.

[3] ZEGPI, E. AND GUTIÉRREZ, S. *Compliance Minimization subject to a Stress Constraint using Small Amplitude Homogenization.* Preprint.

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SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Multiple traces boundary integral formulation for Helmholtz transmission problems

Ralf Hiptmair * Carlos Jerez-Hanckes* †

Abstract

We present a novel boundary integral formulation of the Helmholtz transmission problem for bounded composite scatterers (that is, piecewise constant material parameters in "subdomains") that directly lends itself to operator preconditioning via Calderón projectors. The method relies on local traces on subdomains and weak enforcement of transmission conditions. The variational formulation is set in Cartesian products of standard Dirichlet and special Neumann trace spaces for which restriction and extension by zero operations are well defined. In particular, the Neumann trace spaces over each subdomain boundary are built as piecewise $\widetilde{H}^{-1/2}$-distributions over each associated interface. Through the use of interior Calderón projectors, the problem is cast in variational Galerkin form with an operator matrix whose diagonal is composed of block boundary integral operators associated with the subdomains. We show existence and uniqueness of solutions based on an extension of Lions' projection lemma for non-closed subspaces. We also investigate asymptotic quasi-optimality of conforming boundary element Galerkin discretization. Numerical experiments in 2-D confirm the efficacy of the method and a performance matching that of another widely used boundary element discretization. They also demonstrate its amenability to different types of preconditioning.

References

[1] M. Costabel, *Boundary integral operators on Lipschitz domains: Elementary results*, SIAM J. Math. Anal., 19 (1988), pp. 613–626.

[2] M. Costabel and E. Stephan, *A direct boundary equation method for transmission problems*, J. Math. Anal. Appl., 106 (1985), pp. 367–413.

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[3] R. Hiptmair and C. Jerez-Hanckes , *Multiple Traces Boundary Integral Formulation for Helmholtz Transmission Problems*, SAM Report, ETH Zürich, Seminar for Applied Mathematics, (to appear).

[4] G. C. Hsiao, O. Steinbach, and W. L. Wendland, *Domain decomposition methods via boundary integral equations*, Journal of Computational and Applied Mathematics, 125 (2000), pp. 521 – 537.

[5] A. R. Laliena, M.-L. Rapún, and F.-J. Sayas, *Symmetric boundary integral formulations for Helmholtz transmission problems*, Appl. Numer. Math., 59 (2009), pp. 2814–2823.

[6] U. Langer and O. Steinbach, *Boundary element tearing and interconnecting methods*, Computing, 71 (2003), pp. 205–228.

[7] J.-L. Lions, *Équations différentielles opérationnelles et problèmes aux limites*, Die Grundlehren der mathematischen Wissenschaften, Bd. 111, Springer-Verlag, Berlin, 1961.

[8] W. McLean and O. Steinbach, *Boundary element preconditioners for a hypersingular integral equation on an interval*, Adv. Comput. Math., 11 (1999), pp. 271–286.

[9] T. von Petersdorff, *Boundary integral equations for mixed Dirichlet, Neumann and transmission problems*, Math. Meth. Appl. Sci., 11 (1989), pp. 185–213.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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Numerical homogenization of trabecular bone microstructure using the standard mechanics approach*

ARIEL A. IBARRA† ADRIÁN P. CISILINO **

Resumen

Numerical homogenization is a tool to determine effective macroscopic material properties for microstructured materials. In this work, the standard mechanics approach is used to compute the homogeneous anisotropic linear-elasticity tensor of trabecular bone. Trabecular bone is the material microstructure of bones, which is of enormous biomechanical interest and mainly located inside vertebral bodies and the epiphyses of long bones. It is often affected by osteoporosis in elderly humans. The homogenization procedure is based on a suitable set of microscopic finite element analysis on cubic specimens for macroscopic strain scenarios. The subsequent evaluation of the effective stresses and strains are used to determine homogeneous elasticity tensors. The analyses are performed for artificial periodic microstructures and for representative volume elements of actual trabecular microstructures obtained by means of micro CT scans. The presentation will address the theoretical aspects of the homogenization procedure and the details of its implementation, including the construction of the finite element models from the CT data. There will be discussed the effects of the boundary conditions on the solution.

Referencias

[1] HOLLISTER, S. AND KIKUCHI, N., *A comparison of homogenization and standard mechanics analyses for periodic porous composites.* Computational Mechanics, vol. 10, pp. 73-95, (1992).

[2] IBARRA PINO, A.A. AND CISILINO, A.P., *Determination of effective elastic coefficients using the standard mechanics approach: Application to trabecular bone tissue.* Proccedings of PACAM XI, Foz do Iguaçu, Brasil (2010).

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[3] Ibarra Pino, A.A. and Cisilino, A. P, *El método de homogenización directo aplicado al tejio óseo trabecular*. Proccedings of MECOM IX, Buenos Aires, Argentina (2010).

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Fluid dynamics and unsteady heat transfer in non–Newtonian liquid solidification with natural convection inside cavities*

Nelson O. Moraga †

Abstract

Polymer molding, liquid food freezing, alloy and metals solidification are processes often found in industrial applications. The mathematical model used to describe natural convection in the liquid and mushy zone along heat conduction include continuity, linear momentum and energy non–linear coupled partial differential equations. The moving boundary solidification problem is solved in terms of a temperature dependent liquid phase change fraction. Numerical solutions are obtained by using the Finite Volume Method with the SIMPLE classical segregated algorithm and a novel PSIMPLER one. Cases investigated include conjugated 2D mixed convection / solidification pseudo–plastic non–Newtonian solidification in square cavities and inside the annular space between concentric horizontal cylinders. Dynamic time steps, non uniform staggered grids are used along a new iterative implicit segregated algorithm to solve using successive under–relaxation the governing discrete equations. Unsteady results describing the fluid mechanics and heat transfer include: velocity and temperature distributions, instantaneous location of the liquid–solid moving boundary for different physical applications in food freezing and alloy solidifications. Improvements in calculations speed with the new pressure–velocity coupling algorithm are evaluated in terms of the governing dimensionless parameters.

References

[1] Moraga, N., Riquelme, J., Jauriat, L., *Unsteady conjugate water/air mixed convection in a square cavity.* International Journal of Heat and Mass Transfer, vol. 52, pp. 5512–5524 (2009).

[2] Moraga, N., Andrade, M., Vasco, D., *Unsteady conjugate mixed convection phase change of a power law non–Newtonian fluid in a square cavity.* International Journal of Heat and Mass Transfer, vol. 53, pp. 3308–3318 (2010).

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[3] Moraga, N., Ramrez, S.; Godoy, M.; Ticchione, P., *Study of Convective Non–Newtonian Alloy Solidification in Molds by the PSIMPLER/Finite–Volume Method.* Numerical Heat Transfer, Part A, vol. 57, 12, pp. 936-953, (2010).

[4] Inaba H., Zhang, Y., Horibe, A., Haruki, N., *Numerical simulation of natural convection of latent heat phase–change–material microcapsulate slurry packed in a horizontal rectangular enclosure heated from below and cooled from above.* Heat Mass Transfer, vol. 43, pp. 459-470, (2007).

[5] Assis, E., Katsman, L., Ziskind, G., Letan, R., *Numerical and experimental study of melting in a spherical shell.* International Journal of Heat Mass Transfer, vol. 50, pp. 1790-1804, (2007).

[6] Mat, M.D., Ilegbusi, O.J., *Application of a hybrid model of mushy zone to macrosegregation in alloy solidification.* International Journal of Heat Mass Transfer, vol. 45, pp. 279–289, (2002).

[7] Lin, P., Jaluria, Y., *Heat transfer and solidification of polymer melt flow in a channel.* Polymer Engineering Science, vol. 37, pp. 1247-1258, (2007).

[8] , G., Assimacopoulus, D., *Numerical modeling of convection-diffusion phase phase change problems.* Computational Mechanics, vol. 21, pp. 409-415, (1998).

[9] Giangi, M., Kowalewski, T.A., Stella, F., Leonardi, E., *Natural convection during ice formation: numerical simulation vs. experimental results.* Computer Assisted Mechanic Engineering Science, vol. 7 (3), pp. 321-342, (2000).

[10] Alawadhi, E.M., *Phase change process with free convection in a circular enclosure: numerical simulations.* Computational Fluids, vol. 33, pp. 1335-1348, (2004).

[11] Patankar, S.V., *Numerical Heat Transfer and Fluid Flow.* Hemisphere (Washington), 1980.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Numerical approximation of the spectrum of the curl operator*

Rodolfo Rodríguez † Pablo Venegas†

Abstract

Vector fields $\boldsymbol{H}$ satisfying $\mathbf{curl}\,\boldsymbol{H} = \lambda\boldsymbol{H}$, with λ being a scalar field, are called *force-free fields*. This name arises from magnetohydrodynamics, since a magnetic field of this kind induces a vanishing Lorentz force: $\boldsymbol{F} := \boldsymbol{J} \times \boldsymbol{B} = \mathbf{curl}\,\boldsymbol{H} \times (\mu\boldsymbol{H})$. In 1958 Woltjer [7] showed that the lowest state of magnetic energy density whithin a closed system is attained when λ is spatially constant. In such a case $\boldsymbol{H}$ is called a *linear* force-free field or just a *Trkalian field* [6] and its determination is naturally related with the spectral problem for the curl operator. The eigenfunctions of this problem are known as *free-decay fields* and play an important role, for instance, in the study of turbulence in plasma physics. The spectral problem for the curl operator, $\mathbf{curl}\,\boldsymbol{H} = \lambda\boldsymbol{H}$, has a longstanding tradition in mathematical physics. A large measure of the credit goes to Beltrami [1], who seems to be the first who considered this problem in the context of fluid dynamics and electromagnetism. This is the reason why the corresponding eigenfunctions are also called *Beltrami fields*. On bounded domains, the most natural boundary condition for this problem is $\boldsymbol{H} \cdot \boldsymbol{n} = 0$, which corresponds to a field confined within the domain. Analytical solutions of this problem are only known under particular symmetry assumptions. The first one was obtained in 1957 by Chandrasekhar and Kendall [4] in the context of astrophysical plasmas arising in modeling of the solar crown. More recently, some numerical methods have been introduced to compute force-free fields in domains without symmetry assumptions [2, 3]. In this work, we propose a variational formulation for the spectral problem for the curl operator which, after discretization, leads to a well-posed generalized eigenvalue problem. We propose a method for its numerical solution based on Nédélec finite elements of arbitrary order [5]. We prove spectral convergence, optimal order error estimates and that the method is free of spurious-modes. Finally we report some numerical experiments which confirm the theoretical results and allow us to assess the performance of the method.

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References

[1] E. BELTRAMI, Considerazioni idrodinamiche, *Rend. Inst. Lombardo Acad. Sci. Let.*, **22** (1889) 122–131. (English translation: Considerations on hydrodynamics, *Int. J. Fusion Energy*, **3** (1985) 53–57.)

[2] T. Z. BOULMEZAUD AND T. AMARI, Approximation of linear force-free fields in bounded 3-D domains, *Math. Comp. Model.*, **31** (2000) 109–129.

[3] T. Z. BOULMEZAUD AND T. AMARI, A finite element method for computing nonlinear force-free fields, *Math. Comp. Model.*, **34** (2001) 903–920.

[4] S. CHANDRASEKHAR AND P. C. KENDALL On force-free magnetic fields, *Astrophys. J.*, **126** (1957) 457–460.

[5] J. C. NÉDÉLEC, Mixed finite elements in $\mathbb{R}^3$, *Numer. Math.*, **35** (1980) 315–341.

[6] V. TRKAL, Poznámka k hydrodynamice vazkých tekutin, *Časopis pro Pěstování Mathematiky a Fysiky*, **48** (1919) 302–311.(English translation: A note on the hydrodynamics of viscous fluids, *Czech J. Phys.*, **44** (1994) 97–106.)

[7] L. WOLTJER, *Prod. Natl. Acad. Sci. USA*, **44** (1958) 489–491.

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Energy estimates in semidiscrete time–domain boundary integral equations

Francisco–Javier Sayas *

Abstract

Boundary integral operators in the time domain offer competitive ways to solve exterior scattering problems for different kind of transient (acoustic, elastic, visco–elastic, electromagnetic,...) waves. They also provide exact absorbing boundary conditions that can be placed arbitrarily close to the support of source terms or inhomogeneities. In this talk I will address the problem of how space discretization with Galerkin BEM redistributes the energy, leaking energy to the interior of the scatterer. In the case of transmission problems, BEM–FEM space discretization creates a ghost wave in the interior domain. This effect can be shown by considering abstract wave equations with exotic transmission condition that are exactly satisfied by the semidiscrete equations. This novel energy analysis can be used to better understand the correct balance of energy of Boundary Integral Methods and might lead to evidence for or against the possibility of using non–symmetric BEM–FEM coupled methods in the time domain. Because of its generality, the conclusions are valid both for Galerkin or Convolution Quadrature discretizations in time.

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SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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A coupled system of partial differential and differential algebraic equations for modeling electrical circuits

Mónica Selva-Soto * Caren Tischendorf †

Abstract

During the talk a coupled model for the simulation of electrical circuits will be presented and some of its properties will be discussed. It couples the differential algebraic equations (DAE) that result when modified nodal analysis is applied to an electrical circuit and the partial differential equations (PDE) modeling the behavior of the semiconductor devices in it. The coupling between the DAEs and the PDEs in the model is given in two ways: on one hand the boundary equations for the PDEs in the system depend on the node potentials of the circuit and on the other hand, the current through the semiconductor devices must enter the Kirchoff's current law equations of the circuit. In order to numerically simulate electrical circuits modeled by such a coupled system, we dicretize the PDEs in space and solve the resulting DAE with an appropriate numerical method. Two approaches for the discretization of the PDEs in the system will be considered, finite element methods [2, 3] and mixed finite element methods [1], the properties that the discretized equations have in common will be mentioned and the consequences for the resulting DAEs will be discussed. Some numerical simulations will be shown.

References

[1] Brezzi, F., Marini, L. D., Micheletti, S., Pietra, P., Sacco, R., Wang, S., *Finite element and finite volume discretizations of Drift-Diffusion type fluid models for semiconductors.* In: Handbook of Numerical Analysis, W. H. A. Schilders and E.J.W. ter Maten (Editors), North-Holland-Elsevier Science Publishers B.V., 2002.

[2] Mock, M. S., Analysis of Mathematical Models of Semiconductor Devices (Boole Press), 1983.

[3] Markowich, P. A., The stationary semiconductor device equations (Springer), 1986.

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SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Stabilization of a second order scheme for a GKdV-4 equation modelling surface water waves*

Mauricio Sepúlveda †

Abstract

This work is devoted to the study of a second order numerical scheme for the critical generalized Korteweg-de Vries equation (GKdV with $p = 4$) in a bounded domain. The KdV equation and some of its generalizations as the GKdV type equations appear in Physics, for example in the study of waves on shallow water. Based on the analysis of stability of the first order scheme introduced by Pazoto et al. [1], we add a vanishing numerical viscosity term to a semi-discrete scheme of second order in space so as to preserve similar properties of stability, and thus able to prove the convergence in L^4-strong. The semi-discretization of the spatial structure via second-order central finite difference method yields a stiff system of ODE. Hence, for the temporal discretization, we resort to the two-stage implicit Runge-Kutta scheme of the Gauss-Legendre type. The resulting system is unconditionally stable and possesses favorable nonlinear properties. On the other hand, despite the formation of blow up for the critical case of GKdV, it is known that a localized damping term added to the GKdV-4 equation leads to the exponential decay of the energy for small enough initial conditions, which is interesting from the standpoint of the Control Theory. Then, combining the result of convergence in L^4-strong with discrete multipliers and a contradiction argument, we show that the presence of the vanishing numerical viscosity term allows the uniform (with respect to the mesh size) exponential decay of the total energy associated to the the semi-discrete scheme of higher-order in space with the localized damping term. Numerical experiments are provided to illustrate the performance of the method and to confirm the theoretical results.

References

[1] A. Pazoto, M. Sepúlveda and O.P. Vera Villagrán, Uniform stabilization of numerical schemes for the critical generalized Korteweg-de Vries equation with damping. *Numerische Mathematik* **116, 2** (2010) 317-356.

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[2] M. Sepúlveda, Stabilization of a second order scheme for a GKdV-4 equation modelling surface water waves. Preprint, 2010-02, Departamento de Ingeniería Matemática, Universidad de Concepción, Chile, (2010), submitted.

SANTIAGO NUMÉRICO II

Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Solution procedures for frictional contact

Ernst P. Stephan*

Abstract

The first part of the talk deals with dual formulations for unilateral contact problems with Coulomb friction. Starting from the complementary energy minimization problem, Lagrangian multipliers are introduced to include the governing equation, the symmetry of the stress tensor as well as the boundary conditions on the Neumann and contact boundary. Since the functional arising from the friction part is nondifferentiable an additional Lagrangian multiplier is introduced. This procedure yields a dual-dual formulation of a two-fold saddle point structure. Two different Inf-Sup conditions are introduced to ensure existence of a solution. The system is solved with a nested Uzawa algorithm. In the second part of the talk a mixed hp-time discontinuous Galerkin method for elasto-dynamic contact problem with friction is considered. The contact conditions are resolved by a biorthogonal Lagrange multiplier and are component-wise decoupled. On the one hand the arising problem can be solved by an Uzawa algorithm in conjunction with a block-diagonalization of the global system matrix. On the other hand the decoupled contact conditions can be represented by the problem of finding the root of a non-linear complementary function. This non-linear problem can in turn be solved efficiently by a semi-smooth Newton method. In all cases numerical experiments are given demonstrating the strengths and limitations of the approaches. The talk is based on a joint work with M. Andres and L. Banz.

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SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

A model for blood flow in vessels with discontinuous material properties: exact solutions and numerical methods

Eleuterio F. Toro * Annunziato Siviglia †

Abstract

This work is motivated by two specific medical conditions. The first one concerns abdominal aortic aneuryrisms (AAA), in which mathematical models and numerical simulation tools can play a very useful role in asisiting medical intervention. The second motivation for this work finds its roots in the very recently proposed theory for multiple sclerosis (MS) by Zamboni and collaborators. They associate MS to a vascular condition. Their experimental study would deserve a theoretical study, at least of the heamodynamical aspects reported in their work. Realistic mathematical models for both kinds of problems would invariably make use of hybrid approaches consisting of coupled one-dimensional and three-dimensional submodels. Here formulate a model for the one-dimensional case for blood flow in vessels with discontinuous material properties. The resulting model is a non-linear, non-strictly hyperbolic system with some distinguishing features: nonconservative products are present and nonlinear resonance may take place. The hyperbolic system is analysed in detail and the associated Riemann problems is solved exactly, including resonance. These exact solutions are invaluable for assessing numerical methods intended for more general use. Then we formulate various numerical schemes of the Godunov-type to solve the general initial-boundary value problem. Numerical results are presented and assessed and potential use of the methods for medical applications is discussed.

References

[1] P Zamboni, R Galeotti, E Menegatti, A M Malagoni, G Tacconi, S DallAra, I Bartolomei, F Salvi. *Chronic cerebrospinal venous insufficiency in patients with multiple sclerosis.* J Neurol Neurosurg Psychiatry 2009;80:392399. doi:10.1136/jnnp.2008.157164

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SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Coupled heat and mass transfer in porous media with biological and chemical reactions*

Carlos E. Zambra † Nelson O. Moraga ‡

Abstract

Heat and mass diffusion in self–heating porous media with turbulent air convection is analyzed in this paper. A sequential procedure based on three different mathematical models build on non–linear unsteady coupled partial differential equation is used to describe thermal explotions in compost piles. A $\kappa - \varepsilon$ turbulent air model describes the forced convection around the pile. Arrhenius type source terms are used to assess the biological and chemical effects inside the porous material. Inside the pile, water flow and air diffusion through unsaturated porous media is considered. When the water flow is included, a third source term allows to incorporate the energy balance due to the phase change of water. Time evolution for tempertaure and oxygen concentration is calculated from the coupled energy / mass transfer partial differential equations. The system of governing equations is solved with the finite volume method (FVM) developed by Patankar. A first–order discretization is used to calculate the source, transient and diffusion terms. Convective terms are calculated with the fifth power law as the interpolation function. The SIMPLE algorithm (Semi–Implicit Method for Pressure Linked Equations) is used to calculate, pressure, velocity, temperature in the turbulent air flow around the self–heating porous media. Three computational programs, one for each mathematical model, written in Fortran language, are used to calculated the dependent variables. A suitable combination of the TDMA with the Gauss–Seidel method is implemented as an iterative solver. The solution procedure is applied to describe fluid dynamics, heat and mass transfer processes in two industrial applications: a compost pile and a bio–drying reactor.

References

[1] Versteeg, H.K. and Malalasekera, W., *An Introduction to Computational Fluid Dynamics. The Finite Volume Method.* John Wiley and Sons (New York) , 1995.

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[2] Moraga, N.O., Corvalán, F., Escudey, M., Arias, A., Zambra, C.E., *Unsteady 2D coupled heat and mass transfer in porous media with biological and chemical heat generations*. International Journal of Heat and Mass Transfer, vol. 52, pp. 25–26,(2009).

[3] Escudey, M., Arias. A., Förster, J., Moraga, N., Zambra, C. and Chang. A.C., *Sewage sludge self-heating and spontaneous combustion. Field, Laboratory and numerical studies*. High Temperature Material and Processes, vol. 27, 5, pp. 339–346, (2008).

[4] Patankar, S.V., *Numerical Heat Transfer and Fluid Flow*. Hemisphere (Washington), 1980.

[5] Rada, E.C., Taiss, M., Ragazzi, M., Panaitescu, V. and Apostol, T., *Lower heating value dynamics during municipal solid waste bio–drying*. Environmental Technology, vol. 28, 4, pp. 463–469 (2007).

[6] Richards, L.A, *Capillary conduction of liquids through porous mediums*. Physics, vol. 1, pp. 318–333, (1931).

[7] Serrano, S., *Modeling infiltration with approximated solution to Richards equation*. Journal of Hydrologic Engineering, vol. 9, 5, pp. 421-432, (2004).

[8] Sidhu, H.S., Nelson, M.I., Chen, X.D., *A simple spatial model for self–heating compost piles*. ANZIAM J. (CTAC2006), vol. 41, pp. C135–C150, (2007).

SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

A class of discontinuous Petrov-Galerkin methods: the optimal test norm and time-harmonic wave propagation*

Jeffrey Zitelli † Ignacio Muga ‡ Leszek Demkowicz §
Jayadeep Gopalakrishnan ¶ David Pardo ‖ Victor M. Calo **

Abstract

The phase error, or the pollution effect in the finite element solution of wave propagation problems, is a well known phenomenon that must be confronted when solving problems in the high-frequency range. This paper presents a new method with *no phase errors* for one-dimensional (1D) time-harmonic wave propagation problems using new ideas that hold promise for the multidimensional case. The method is constructed within the framework of the Discontinuous Petrov-Galerkin (DPG) method with optimal test functions. These methods select solutions that are the best possible approximations in an energy norm dual to any selected test space norm. Thus, we advance by asking what is the *optimal test space norm* that achieves error reduction in a given energy norm. This is answered in the specific case of the Helmholtz equation with L^2-norm as the energy norm. We obtain uniform stability with respect to the wave number. We illustrate the method with a number of 1D numerical experiments, using discontinuous piecewise polynomial *hp* spaces for the trial space and its corresponding optimal test functions computed approximately and locally. A 1D theoretical stability analysis is also developed.

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References

[1] Ivo M. Babuska and Stefan A. Sauter. Is the pollution effect of the FEM avoidable for the helmholtz equation considering high wave numbers? *SIAM J. Numer. Anal.*, 34(6):2392–2423, 1997.

[2] P. E. Barbone and I. Harari. Nearly H^1-optimal finite element methods. *Comput. Methods Appl. Mech. Engrg.*, 190:5679 – 5690, 2001.

[3] C.L. Bottasso, S. Micheletti, and R. Sacco. The discontinuous Petrov-Galerkin method for elliptic problems. *Comput. Methods Appl. Mech. Engrg.*, 191:3391–3409, 2002.

[4] L. Demkowicz and J. Gopalakrishnan. A class of discontinuous Petrov-Galerkin methods. Part II: Optimal test functions. Technical Report 16, ICES, 2009. In print (*Numer. Methods Partial Differential Equations*).

[5] X. Feng and H. Wu. Discontinuous Galerkin methods for the Helmholtz equation with large wave number. *SIAM J. Numer. Anal.*, 47:2872 – 2896, 2009.

[6] Isaac Harari. A survey of finite element methods for time-harmonic acoustics. *Computer Methods in Applied Mechanics and Engineering*, 195(13-16):1594 – 1607, 2006. A Tribute to Thomas J.R. Hughes on the Occasion of his 60th Birthday.

[7] A. F. D. Loula and D. T. Fernandes. A quasi optimal Petrov-Galerkin method for Helmholtz problem. *Internat. J. Numer. Methods Engrg.*, 80:1595 – 1622, 2009.

[8] Assad A. Oberai and Peter M. Pinsky. A numerical comparison of finite element methods for the Helmholtz equation. *Journal of Computational Acoustics*, 8(1):211, 2000.

[9] Lonny L. Thompson. A review of finite-element methods for time-harmonic acoustics. *The Journal of the Acoustical Society of America*, 119(3):1315–1330, 2006.

[10] J. Zitelli, I. Muga, L. Demkowicz, J. Gopalakrishnan, D. Pardo, V.M. Calo. A class of discontinuous Petrov-Galerkin methods. Part IV: Wave propagation. Technical Report 17, ICES, 2010. In review.