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A model for blood flow in vessels with discontinuous material properties: exact solutions and numerical methods.

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Abstract

This work is motivated by two specific medical conditions. The first one concerns abdominal aortic aneuryrisms (AAA), in which mathematical models and numerical simulation tools can play a very useful role in asisiting medical intervention. The second motivation for this work finds its roots in the very recently proposed theory for multiple sclerosis (MS) by Zamboni and collaborators. They associate MS to a vascular condition. Their experimental study would deserve a theoretical study, at least of the heamodynamical aspects reported in their work.

Realistic mathematical models for both kinds of problems would invariably make use of hybrid approaches consisting of coupled one-dimensional and three-dimensional submodels. Here formulate a model for the one-dimensional case for blood flow in vessels with discontinuous material properties. The resulting model is a non-linear, non-strictly hyperbolic system with some distinguishing features: nonconservative products are present and nonlinear resonance may take place. The hyperbolic system is analysed in detail and the associated Riemann problems is solved exactly, including resonance. These exact solutions are invaluable for assessing numerical methods intended for more general use.

Then we formulate various numerical schemes of the Godunov-type to solve the general initial-boundary value problem. Numerical results are presented and assessed and potential use of the methods for medical applications is discussed.

References

[1] P Zamboni, R Galeotti, E Menegatti, A M Malagoni, G Tacconi, S DallAra, I Bartolomei, F Salvi. *Chronic cerebrospinal venous insufficiency in patients with multiple sclerosis*. J Neurol Neurosurg Psychiatry 2009;80:392399. doi:10.1136/jnnp.2008.157164

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