SANTIAGO NUMÉRICO II
Quinto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Diciembre 9–11, 2010

Analysis of an augmented mixed finite element method for the Stokes-Darcy coupled problem*

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Abstract

In this paper we analyze an augmented mixed finite element method for the coupling of fluid flow with porous media flow. Flows are governed by the Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider a semi-augmented mixed formulation, augmented in the Stokes domain and dual-mixed in the Darcy region, which yields a compact perturbation of an invertible mapping as the resulting operator equation. The approach, which extends recent results on the a priori and a posteriori error analysis of a fully-mixed formulation for the Stokes-Darcy model, is based on the introduction of the Galerkin least-squares type terms arising from the constitutive and equilibrium equations of the Stokes equation, and from the relations defining the free fluid pressure in terms of the stress tensor and the vorticity in terms of the free fluid velocity. All these terms are multiplied by stabilization parameters that can be chosen so that the resulting continuous formulation becomes well posed. We then apply a classical result on projection methods for Fredholm operators of index zero to show, under suitable hypotheses on the finite element subspaces for the Darcy region, that the use of arbitrary finite element subspaces for the Stokes domain implies the well-posedness of the corresponding augmented Stokes-Darcy Galerkin scheme. Next, we derive a reliable and efficient residual-based a posteriori error estimator for the augmented mixed finite element scheme.

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

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