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Multiple Traces Boundary Integral Formulation for Helmholtz Transmission Problems

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Abstract

We present a novel boundary integral formulation of the Helmholtz transmission problem for bounded composite scatterers (that is, piecewise constant material parameters in "subdomains") that directly lends itself to operator preconditioning via Calderón projectors. The method relies on local traces on subdomains and weak enforcement of transmission conditions. The variational formulation is set in Cartesian products of standard Dirichlet and special Neumann trace spaces for which restriction and extension by zero operations are well defined. In particular, the Neumann trace spaces over each subdomain boundary are built as piecewise $\widetilde{H}^{-1/2}$-distributions over each associated interface. Through the use of interior Calderón projectors, the problem is cast in variational Galerkin form with an operator matrix whose diagonal is composed of block boundary integral operators associated with the subdomains. We show existence and uniqueness of solutions based on an extension of Lions' projection lemma for non-closed subspaces. We also investigate asymptotic quasi-optimality of conforming boundary element Galerkin discretization. Numerical experiments in 2-D confirm the efficacy of the method and a performance matching that of another widely used boundary element discretization. They also demonstrate its amenability to different types of preconditioning.

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