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A velocity-pseudostress formulation for a class of quasi-Newtonian Stokes flows*

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Abstract

In this paper we introduce and analyze new mixed finite element schemes for a class of nonlinear Stokes models arising in quasi-Newtonian fluids. The methods are based on a non-standard mixed approach in which the velocity, the pressure, and the pseudostress are the original unknowns. However, we use the incompressibility condition to eliminate the pressure, and set the velocity gradient as an auxiliary unknown, which yields a twofold saddle point operator equation as the resulting dual-mixed variational formulation. In addition, a suitable augmented version of the latter showing a single saddle point structure is also considered. We apply known results from nonlinear functional analysis to prove that the corresponding continuous and discrete schemes are well-posed. In particular, we show that Raviart-Thomas elements of order $k \geq 0$ for the pseudostress, and piecewise polynomials of degree k for the velocity and its gradient, ensure the well-posedness of the associated Galerkin schemes. Moreover, we prove that any finite element subspace of the square integrable tensors can be employed to approximate the velocity gradient in the case of the augmented formulation. Then, we derive a reliable and efficient residual-based a posteriori error estimator for each scheme. Finally, we provide several numerical results illustrating the good performance of the resulting mixed finite element methods, confirming the theoretical properties of the estimator, and showing the behaviour of the associated adaptive algorithms.

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