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Riemann Solution for the Three-Phase Injection Problem in Virgin Reservoirs with General Relative Permeabilities

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Abstract

We consider the Riemann problem for two conservation laws representing the injection of two fluids into a virgin reservoir containing a third fluid. We assume that the three fluids are immiscible and do not exchange mass. The relative permeabilities are given either by the Corey or Stone models as general saturation powers. For each proportion of the injected fluids the Riemann solution profile and the wave curves are found.

The solution consists constant states, rarefaction and shock waves. Typically there are two wave groups: the slower one is composed by a rarefaction adjacent to a shock, in which the three fluid saturations change. Depending on the initial conditions, a constant state can exist between the two wave groups. In the faster wave group there are only two fluids, and it consists either of a rarefaction adjacent to a shock or of a single shock.

The Riemann solution that maximizes useful oil recovery is represented by a single wave group involving the three fluids, and it consists of a rarefaction adjacent to a shock wave, leading to a shock that is faster than in the other cases. We also observe that both models give qualitatively the same solutions profiles, despite the presence of an elliptic region in Stone model.

In particular we determine, for any choice of the phase viscosities and of the permeability expressions, the proportion of the injected fluids that maximizes the magnitude and speed of the principal shock, which is responsible for the recovery at breakthrough. The three-phase wave curve in this Riemann solution and the location of the umbilic point (or the elliptic region) are correlated. The wave curve in the saturation triangle and its profile are found numerically, leading to the construction of Riemann solutions for Corey and Stone models for different permeabilities.

The Riemann construction of solutions for the general Corey or Stone models can be used to improve the results in oil recovery modeling. We demonstrate the well-posedness of the solutions for small parameter deviations of permeability powers or viscosities.

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