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On nonlocal conservation laws modeling sedimentation*

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Abstract

The well-known kinematic sedimentation model by Kynch states that the settling velocity of small equal-sized particles in a viscous fluid is a function of the local solids volume fraction. This assumption converts the one-dimensional solids continuity equation into a scalar, nonlinear conservation law with a non-convex and local flux. The present work deals with a modification of this model, and is based on the assumption that either the solids phase velocity or the solid-fluid relative velocity at a given position and time depends on the concentration in a neighborhood via convolution with a symmetric kernel function with finite support. This assumption is justified by theoretical arguments arising from stochastic sedimentation models, and leads to a conservation law with a nonlocal flux. The alternatives of velocities for which the nonlocality assumption can be stated lead to different algebraic expressions for the factor that multiplies the nonlocal flux term. In all cases, solutions are in general discontinuous and need to be defined as entropy solutions. An entropy solution concept is introduced, jump conditions are derived and uniqueness of entropy solutions in shown. Existence of entropy solutions is established by proving convergence of a difference-quadrature scheme. It turns out that only for the assumption of nonlocality for the relative velocity it is ensured that solutions of the nonlocal equation assume physically relevant solution values between zero and one. Numerical examples illustrate the behaviour of entropy solutions of the nonlocal equation.

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