A dual-mixed analysis for incompressible quasi-Newtonian flows[*]

Mauricio A. Barrientos [†]   Matthias Maischak [‡]

Abstract

We consider the coupling of dual-mixed finite element method and boundary integral equation method to solve a transmission problem between a lineal Stokes flow with a quasi-Newtonian flow with mixed boundary conditions . The result is a new mixed scheme for the quasi-Newtonian problem. The approach is based on the introduction of both the flux and the strain tensor as futher unknows, wich yields a two-fold saddle point operator equation as the resulting variational formulation. We derive existence and uniqueness of solution for the continuous and discrete formulations and provide the associated error analysis. In particular, the corresponding Galerkin scheme is defined by using piecewise constant functions and Raviar-Thomas spaces of lowest order. Most of our analysis makes use of an extension of the classical Babuska-Brezzi theory to a class of nonlinear saddle-point problems. Also, we develop a-posteriori error estimates (based on Bank-Weiser type) and propose and reliable adaptive algorithm to compute the finite elements solutions. Finally, several numerical results are provided.

References

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[†]Instituto de Matemáticas, Facultad de Ciencias, Pontificia Universidad Católica de Valparaíso, Valparaíso, Chile, e-mail: `mauricio.barrientos@ucv.cl`

[‡]BICOM, Brunel University, Uxbridge, UB8 3PH, UK. e-mail: `matthias.maischak@brunel.ac.uk`

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